

6. minitest - varianta A

Integrál funkce jedné proměnné

11. 4. 2024

Vypočítejte integrály

$$a) \int_0^1 \sqrt{x}(x-1)^2 dx \quad b) \int \frac{1}{x \ln^3 x} dx$$

$$\begin{aligned}
 a) \int_0^1 \sqrt{x}(x-1)^2 dx &= \int_0^1 \sqrt{x}(x^2 - 2x + 1) dx = \int_0^1 \left(x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + x^{\frac{1}{2}} \right) dx \\
 &= \left[\frac{x^{\frac{7}{2}}}{\frac{7}{2}} - 2 \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 = \frac{2}{7} - \frac{4}{5} + \frac{2}{3} = \frac{30 - 84 + 70}{105} = \\
 &= \frac{16}{105}
 \end{aligned}$$

$$\begin{aligned}
 b) \int \frac{1}{x \ln^3 x} dx &= \int \frac{1}{t^3} dt = \int t^{-3} dt = \frac{t^{-2}}{-2} + C = \\
 &= -\frac{1}{2 \ln^2 x} + C
 \end{aligned}$$

$\left. \begin{array}{l} \ln x = t \\ \frac{1}{x} dx = dt \end{array} \right\}$

6. minitest - varianta B

Integrál funkce jedné proměnné

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Vypočtete integrály

a) $\int_0^1 x(\sqrt{x}-1)^2 dx$ b) $\int \frac{e^x}{e^x+1} dx$

$$\begin{aligned} \text{a) } \int_0^1 x(\sqrt{x}-1)^2 dx &= \int_0^1 x(x-2\sqrt{x}+1) dx = \int_0^1 (x^2-2x^{\frac{3}{2}}+x) dx \\ &= \left[\frac{x^3}{3} - 2 \cdot \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^2}{2} \right]_0^1 = \frac{1}{3} - 2 \cdot \frac{2}{5} + \frac{1}{2} = \frac{10-24+15}{30} = \underline{\underline{\frac{1}{30}}} \end{aligned}$$

$$\begin{aligned} \text{b) } \int \frac{e^x}{e^x+1} dx &= \int \frac{dt}{t} = \ln|t| + c = \underline{\underline{\ln(e^x+1) + c}} \\ &\left. \begin{array}{l} e^x+1 = t \\ e^x dx = dt \end{array} \right\} \end{aligned}$$