8. minitest - varianta A

Integrace per-partes

25. 4. 2024

Vypočtěte integrál

$$\int_0^{\infty} x\, e^{-3x}\, dx$$

$$\int_0^{\infty} x e^{-3x}\, dx \overset{P.P.}{=} \left[x \cdot \left(-\frac{1}{3} e^{-3x}\right) \right]_0^{\infty} + \int_0^{\infty} \frac{1}{3} e^{-3x}\, dx =$$

$$\left| \begin{matrix} u = x & v' = e^{-3x} \\ u' = 1 & v = -\frac{1}{3} e^{-3x} \end{matrix} \right|$$

$$\boxed{\int u v' = u \cdot v - \int u' v}$$

$$= \underbrace{\lim_{x \to +\infty} \left(-\frac{x}{3e^{3x}} \right)} - 0 + \left[-\frac{1}{9} e^{-3x} \right]_0^{\infty}$$

$$\overset{L.P.}{=} \lim_{x \to \infty} \left(-\frac{1}{9e^{3x}} \right) = 0$$

$$= 0 + \underbrace{\lim_{x \to \infty} \left(-\frac{1}{9} e^{-3x} \right)}_{0} + \frac{1}{9} e^{0} = \underline{\underline{\frac{1}{9}}}$$

8. minitest - varianta B

Integrace per-partes

25. 4. 2024

Vypočtěte integrál

$$\int_0^\infty x\, e^{-2x}\, dx$$

$$\int x e^{-2x}\, dx \overset{P.P.}{=} -\frac{1}{2} x e^{-2x} + \frac{1}{2}\int e^{-2x}\, dx = -\frac{x}{2e^{2x}} - \frac{1}{4e^{2x}} + c$$

$$\left| \begin{matrix} u = x & v' = e^{-2x} \\ u' = 1 & v = -\frac{1}{2} e^{-2x} \end{matrix} \right| \qquad \boxed{\int u v' = u \cdot v - \int u' v}$$

$$\int_0^\infty x e^{-2x}\, dx = \left[-\frac{x}{2e^{2x}} - \frac{1}{4e^{2x}} \right]_0^\infty = \lim_{x \to \infty} \left(-\frac{x}{2e^{2x}} - \frac{1}{4e^{2x}} \right)$$

$$- \left(-0 - \frac{1}{4} \right) = \underbrace{\lim_{x\to\infty} \left(-\frac{x}{2e^{2x}} \right)}_{\overset{L.P.}{=} \lim \left(-\frac{1}{4e^{2x}} \right) = 0} - \underbrace{\lim_{x \to \infty} \frac{1}{4e^{2x}}}_{0} + \frac{1}{4}$$

$$= \frac{1}{4}$$