$$f(x)=(x^2+1)\cdot e^{-|x+1|} = \begin{cases}(x^2+1)\cdot e^{x+1}, & x\in(-\infty,-1)\\ (x^2+1)\cdot e^{-(x+1)}, & x\in\langle -1,\infty)\end{cases}$$

I. $x\in(-\infty,-1)$: $f(x)=(x^2+1)\cdot e^{x+1}$

$$f'(x)=2x\cdot e^{x+1}+(x^2+1)e^{x+1}=e^{x+1}(x^2+2x+1)$$

$$=e^{x+1}\cdot(x+1)^2>0 \quad \forall x\in(-\infty,-1)$$

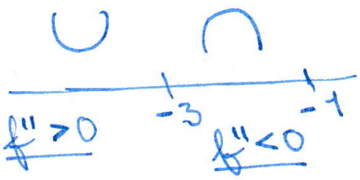
$\Rightarrow$ funkce je rostoucí na $(-\infty,-1)$

$$f''(x)=e^{x+1}\cdot(x^2+2x+1)+e^{x+1}\cdot(2x+2)=e^{x+1}(x^2+4x+3)$$

$$f''(x)=0 \iff x^2+4x+3=0$$

$$(x+3)(x+1)=0$$

$$x=-3 \lor x=-1$$



INFLEXNÍ BOD

$\forall x\in(-\infty,-3)$: $f''(x)>0 \Rightarrow$ funkce je konvexní

$\forall x\in(-3,-1)$: $f''(x)<0 \Rightarrow$ funkce je konkávní

$$\lim_{x\to-\infty}(x^2+1)\cdot e^{x+1}=\lim_{x\to+\infty}((-x)^2+1)\cdot e^{-x+1}=e\cdot\lim_{x\to\infty}\frac{x^2+1}{e^x}=\underline{\underline{0}}$$

II. $x\in\langle-1,\infty)$: $f(x)=(x^2+1)\cdot e^{-(x+1)}$

$$f'(x)=2x\cdot e^{-(x+1)}-(x^2+1)\cdot e^{-(x+1)}=-e^{-(x+1)}\cdot(x-1)^2$$

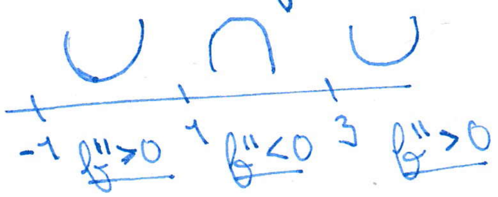
$\forall x\in\langle-1,\infty)$: $f'(x)\geq 0 \Rightarrow$ funkce je nerostoucí

$$f''(x)=e^{-(x+1)}\cdot(x-1)^2+(-e^{-(x+1)}\cdot 2(x-1))=e^{-(x-1)}\cdot(x^2-4x+3)$$

$$f''(x)=0 \iff x^2-4x+3=0$$

$$(x-3)(x-1)=0$$

$$x=3 \lor x=1$$



INFLEXNÍ BODY

$\forall x\in(-1,1)\cup(3,\infty)$: $f''(x)>0 \Rightarrow$ funkce je konvexní

$\forall x\in(1,3)$: $f''(x)<0 \Rightarrow$ funkce je konkávní

$$\lim_{x\to+\infty}(x^2+1)e^{-(x+1)}=\lim_{x\to\infty}\frac{x^2+1}{e^{x+1}}=\underline{\underline{0}}$$

Jednostranné derivace v bodě $x = -1$:

$$f'_+(-1) = \lim_{x \to -1_+} f'(x) = \lim_{x \to -1_+} \left(-e^{-(x+1)} \cdot (x-1)^2\right) = -4$$

$$f'_-(-1) = \lim_{x \to -1_-} f'(x) = \lim_{x \to -1_-} \left(e^{x+1} \cdot (x+1)^2\right) = 0$$

obor hodnot:

$H_f = (0, 2\rangle$

x	-1	1	3	-3	0
$f(x)$	2	$2e^{-2}$	$10e^{-4}$	$10e^{-2}$	e^{-1}