

2. minitest MA1

Varianta B
18. 10. 2024

Určete definiční obor a obor hodnot dané funkce. Rozhodněte, zda je funkce prostá a pokud ano, určete předpis funkce inverzní.

$$f(x) = \arccos(\log_2 \sqrt[3]{x})$$

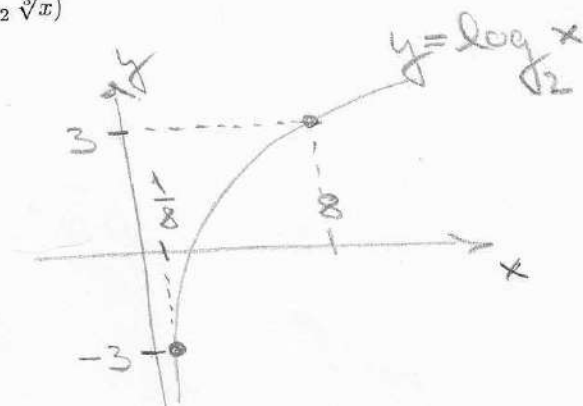
$$D_f: \quad -1 \leq \log_2 \sqrt[3]{x} \leq 1$$

$$-1 \leq \frac{1}{3} \log_2 x \leq 1 \quad | \cdot 3$$

$$-3 \leq \log_2 x \leq 3$$

$$\frac{1}{8} = 2^{-3} \leq x \leq 2^3 = 8$$

$\wedge x > 0$

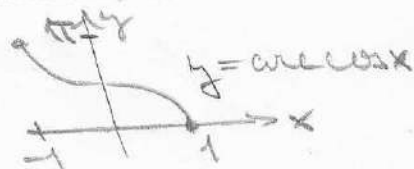


$$D_f = \left\langle \frac{1}{8}, 8 \right\rangle$$

$$f(x) = (f_2 \circ f_1)(x)$$

$$f_1(x) = \frac{1}{3} \log_2 x$$

$$f_2(x) = \arccos x$$



$$\left\langle \frac{1}{8}, 8 \right\rangle \xrightarrow{f_1} \langle -1, 1 \rangle \xrightarrow{f_2} \langle 0, \pi \rangle = H_f$$

$$f^{-1}: \quad x = \arccos(\log_2 \sqrt[3]{y})$$

$$\cos x = \frac{1}{3} \log_2 y$$

$$3 \cos x = \log_2 y$$

$$f^{-1}: \quad y = 2^{3 \cos x} = 8^{\cos x}, \quad x \in \langle 0, \pi \rangle$$

Varianta A

$$f(x) = \arccos\left(\frac{3x-1}{6x+5}\right)$$

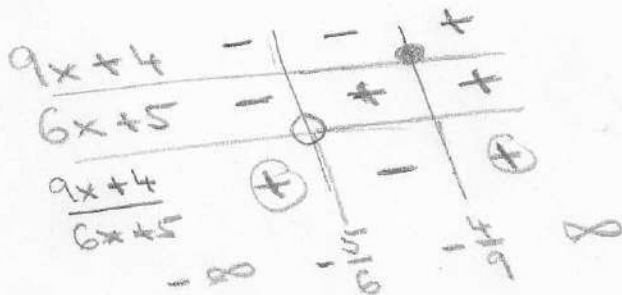
$$D_f: -1 \leq \frac{3x-1}{6x+5} \leq 1$$

$$I. -1 \leq \frac{3x-1}{6x+5}$$

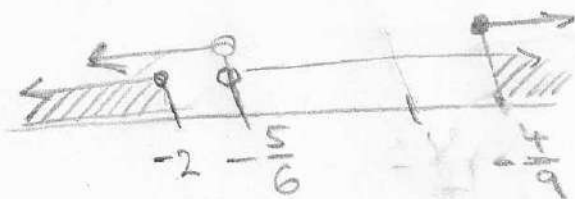
$$0 \leq \frac{3x-1}{6x+5} + 1$$

$$0 \leq \frac{3x-1+6x+5}{6x+5}$$

$$0 \leq \frac{9x+4}{6x+5}$$



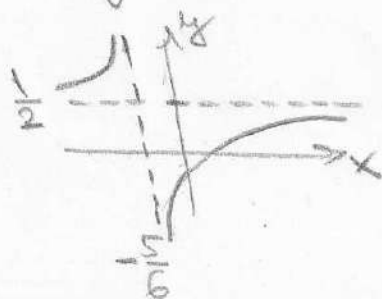
$$x \in (-\infty, -\frac{5}{6}) \cup (-\frac{4}{9}, \infty)$$



$$f(x) = (b_2 \circ b_1)(x)$$

$$b_1(x) = \frac{3x-1}{6x+5}$$

$$b_2(x) = \arccos x$$



$$D_f \xrightarrow{b_1} \langle -1, 1 \rangle \setminus \left\{ \frac{1}{2} \right\} \xrightarrow{b_2} \langle 0, \pi \rangle \setminus \left\{ \frac{\pi}{3} \right\}$$

$$\text{nebot } \arccos \frac{1}{2} = \frac{\pi}{3}$$

Obez hodnot

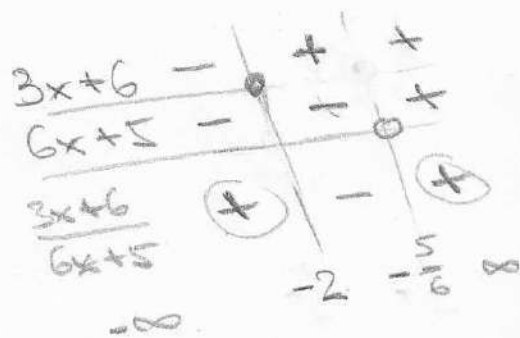
$$| H_f = \langle 0, \pi \rangle \setminus \left\{ \frac{\pi}{3} \right\} |$$

$$II. \frac{3x-1}{6x+5} \leq 1$$

$$\frac{3x-1-(6x+5)}{6x+5} \leq 0$$

$$\frac{-3x-6}{6x+5} \leq 0 \quad (\cdot (-1))$$

$$\frac{3x+6}{6x+5} \geq 0$$



$$x \in (-\infty, -2) \cup (-\frac{5}{6}, \infty)$$

$$D_f = (-\infty, -2) \cup (-\frac{4}{9}, \infty)$$

$$f^{-1}: x = \arccos\left(\frac{3y-1}{6y+5}\right)$$

$$\cos x = \frac{3y-1}{6y+5}$$

$$(\cos x) \cdot (6y+5) = 3y-1$$

$$6y \cos x + 5 \cos x = 3y - 1$$

$$6y \cos x - 3y = -1 - 5 \cos x$$

$$y(6 \cos x - 3) = -1 - 5 \cos x$$

$$f^{-1}: y = \frac{-1 - 5 \cos x}{6 \cos x - 3}, \quad x \in \langle 0, \pi \rangle - \left\{ \frac{\pi}{3} \right\}$$